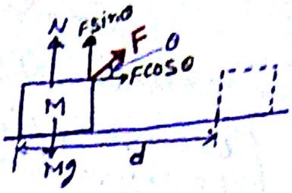
'WORK. POWER. ENERY'

WORK (W)

I → If force const.

$$\boxed{W = (F\cos\theta)d = \vec{F}\cdot\vec{d}}$$

$\vec{d}$ = displacement.
θ = Angle b/w Force & disp.



$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$
$\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\boxed{W = \vec{F}\cdot\vec{d} = F_x x + F_y y + F_z z}$$

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

* Initial position vector
$\boxed{\vec{r_i} = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}}$

* Final position vector
$\boxed{\vec{r_f} = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}}$

* Disp. vector
$\vec{d} = \vec{r_f} - \vec{r_i} = (x_2-x_1)\hat{i} + (y_2-y_1)\hat{j} + (z_2-z_1)\hat{k}$

$$\boxed{W = \vec{F}\cdot\vec{d} = F_x(x_2-x_1) + F_y(y_2-y_1) + F_z(z_2-z_1)}$$

$\vec{F_1}, \vec{F_2} - - - \vec{F_N}$

$W = W_1 + W_2 + - - - W_N$

$$\boxed{W = \vec{F_1}\cdot\vec{d_1} + \vec{F}\cdot\vec{d_2} + - - - \vec{F_N}\cdot\vec{d_N}}$$

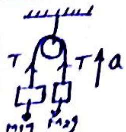
$\boxed{W = Fd\cos\theta}$

$W = \oplus ve$
$0° < \theta < 90°$

$W = \ominus ve$
$90° < \theta < 180°$

$W = 0$
$\theta = 90$
or
$F = 0$
or
$d = 0$

Ex → $M_1 > M_2$

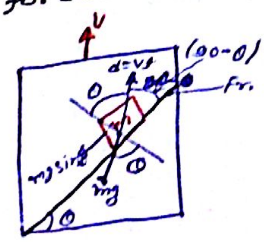


* Work done by gravitational force on mass –
$M_1 = W_1 = (M_1 g)d\cos 0° = \oplus ve$
$M_2 = W_2 = (M_2 g)d\cos 180° = \ominus ve$

* Work done by tension force on mass.
$M_1 = W_1 = T\cdot d\cos 180° = \ominus ve$
$M_2 = W_2 = T\cdot d\cos 0° = \oplus ve$.

Eg → * Work done by centripetal force in circular path.
* Work done on charge particle in presence of uniform magnetic field.
* Work done by collie.
* Work done by tension force in a simple pendulum.

Particle of Mass 'M' placed on Incline plane wich is move upward with const velocity. If mass 'M' Remain at Rest w.r.t incline plane. Work done by gravitational force on mass 'M' after time 't'.



* Work done by Friction force
$W = F_r(vt)\cos(90-\theta)$
* If body ⊕nt at Rest
$\boxed{F_L > mg\sin\theta}$
$\boxed{W = (mg\,vt)\sin^2\theta}$

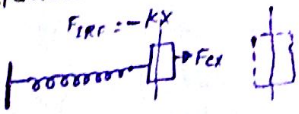
* Work done by gravitational force
$W = (Fg)(vt)\cos 180°$
$= (mg)(vt)(-1)$
$\boxed{W_g = -(mg\,vt)}$

II → If Force is variable.

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$] (x_1, y_1, z_1)
$\vec{dr} = dx\hat{i} + F_y\hat{j} + dz\hat{k}$] (x_2, y_2, z_2)

$$\boxed{W = \vec{F}\cdot\vec{dr} = \int_{x_1}^{x_2} F_x\,dx + \int_{y_1}^{y_2} F_y\,dy + \int_{z_1}^{z_2} F_z\,dz}$$

Internal Resolving force of spring is change w.r.t distance as Relation $F = -kx$. Work done by the applied force to displace the spring x_1 to x_2.



$F_{ex} = -F_{IRF} = -(-kx)$
$\boxed{F_x = kx}$

* Force compression, extended.
$$\boxed{W = \frac{1}{2}(x_2^2 - x_1)^2}$$

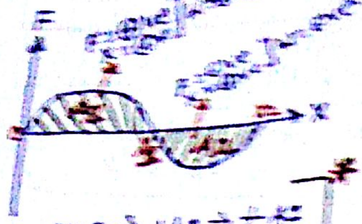
$x_1 = 0,\ x_2 = x$
$$\boxed{W = \frac{1}{2}kx^2}$$

Work Done From Force, Disp curve

* Area inclosed b/w Force disp curve & disp axis Represent Work done by by the force.

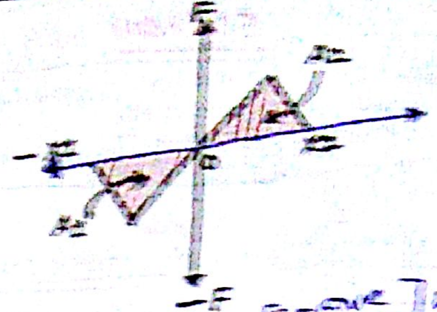
$$\boxed{W = \int_{x_1}^{x_2} F\cdot dx = \text{Area inclosed b/w "F-x" curve \& x-axis.}}$$

#



[* $0 < x < x_1 \Rightarrow W_1 \Rightarrow +ve$
* $x_1 < x < x_2 \Rightarrow W_2 \Rightarrow -ve$
* $0 < x < x_2 \Rightarrow W = W_1 - W_2$] *

#



[* $0 < x < x_1$ [F = [illegible], x = [illegible]] W = [illegible]
* $0 < x < x_2$ [F = [illegible], x = [illegible]] W = [illegible]] *

Conservative & Non-conservative Work

- **Conservative Work** → * Work done is independent from path is only depend on initial & Final point.

$\boxed{W_1 = W_2 = W_3}$

* If Work done in a closed path is zero then it's done by conservative force.

Ex → Gravitational Force
* Electrostatic Force
* Internal Restoring Force
* Force by bar magnet.

- **Non-conservative Force** → Work done depend on path b/w initial & final point.

$\boxed{W_1 \neq W_2 \neq W_3}$

* In a close path Work done is not equal to zero.

Ex → Friction Force
* Viscous Force
| * Air Resistance
| * Magnetic force field of current carrying wire.

POWER(P)

Capacity of Work done per unit time.

$$\boxed{P_{avg} = \frac{\Delta W}{\Delta t}}$$

$$\boxed{P_{ins} = \frac{dW}{dt} = \frac{\vec{F}\cdot d\vec{x}}{dt} = \vec{F}\cdot\vec{V}}$$ (Linear motion)

$$\boxed{P = \vec{\tau}\cdot\vec{\omega}}$$ (Angular motion)

* Scalar quantity
* SI unit = watt (W)
[1 W = 1 J/sec]
* 1 HP = 746 watt.

NOTE → * Slope of Work-time curve Represent power of device.

$$\boxed{\text{Slope} = \frac{dy}{dx} = \frac{dW}{dt} = P = \tan\theta}$$ *

$P = \tan\theta$ ✓

$\boxed{\tan\theta_2 > \tan\theta_1} \Rightarrow P_2 > P_1$ ✓

* Area Inclosed b/w power-time curve & time represent Work done of device.

$$\boxed{W = \int P\,dt = \text{Area Inclosed b/w P-T curve \& time axis}}$$ *

$\boxed{W_{net} = W_1 + W_2}$

In a hydraulic power plant electrical power delivered by power plant.

$$\boxed{P_e = \eta\left(\frac{mgh}{t}\right)}$$ η = Efficiency, P_e = Electrical power.

✓ If a Wind turbine wind energy converted in a electrical energy. If velocity of wind is V & If efficiency of generator is 100% Then electrical power proportional to

$$\boxed{\eta = 100\% = \frac{P_{out}}{P_{in}}}$$ $\boxed{P_{out} = P_{in}}$ * $\boxed{P \propto V^3}$ *

ENERGY

* Scalar quantity
* Unit → SI → Joule; CGS → Erg

$P.E = \ominus ve$, Force $= \ominus ve$] → Attractive force

$PE = \oplus ve$, Force $= \oplus ve$] Repulsive force

* $1J = 10^7$ erg
* $1 cal = 4.2J$
* $1 eV = 1.6 \times 10^{-19} J$
* $1 kWh = 10^3 erg (J/sec)(3600 sec)$
  $= 3.6 \times 10^6 J$

* Mechanical Energy (M.E)

$$\boxed{M.E = K.E + P.E}$$

[A] → * Kinetic Energy (K.E)

$$\boxed{K.E = \frac{1}{2} m v^2}$$

v → Linear velocity of particle.

m → Mass of particle = Inertia of Linear motion.

Translational K.E

* Momentum (P)

$$\boxed{P = mv}$$

$$\boxed{K.E = \frac{1}{2}mv^2 = \frac{P^2}{2m} = \frac{1}{2}PV}$$

AIR AIIMS

* $\vec{P} = 0$, K.E may be zero.

(m) $\xrightarrow{+P}$ $\xleftarrow{-\vec{P}}$ (m)

$\vec{P}_{net} = 0$

$$\boxed{K.E = \frac{P^2}{2m} + \frac{P^2}{2m} \neq 0}$$

* If $K.E = 0 \Rightarrow P \Rightarrow$ must be zero.
* If $\vec{P}$ change ⇒ Then K.E may be change.
* If K.E is change ⇒ Then $\vec{P}$ must change.

* concept

$$y = \frac{x^a y^b}{p^q} = x^a y^b p^{-q}$$

$a, b, q =$ const.

$$\frac{\Delta y}{y} = a\left(\frac{\Delta x}{x}\right) + b\left(\frac{\Delta z}{z}\right) + (-q)\left(\frac{\Delta p}{p}\right)$$

[B] → Potential Energy (P.E)

* Energy Related to piston or, configuration.
* P.E is Related to only for conservative force.
* P.E is Related to only for conservative force field store in the form of P.E in a system.
* Work done against conservative force

$$\boxed{\Delta U = -\Delta W_C}$$

$$\boxed{\Delta U = -F \Delta r \cos\theta}$$

$\theta = 0 \Rightarrow \Delta U = -F\Delta r$

$$\boxed{F = -\frac{\Delta U}{\Delta r}}$$

$F = f(x, y, z)$

$$\int du = -\int F \cdot dr$$

$$\boxed{du = \left[\int F_x dx + \int F_y dy + \int F_z dz\right]}$$

$U = f(x, y, z)$

$F_x = -\left(\frac{du}{dx}\right)$ y, z = const.

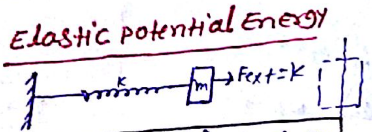
$F_y = -\left(\frac{dU}{dy}\right)$ x, y = const.

$F_z = \left(-\frac{dU}{dz}\right)$ xy = const.

$$\boxed{\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}}$$

$$\boxed{|\vec{F}| = \sqrt{F_x^2 + F_y^2 + F_z^2}}$$

|i| → Elastic potential Energy

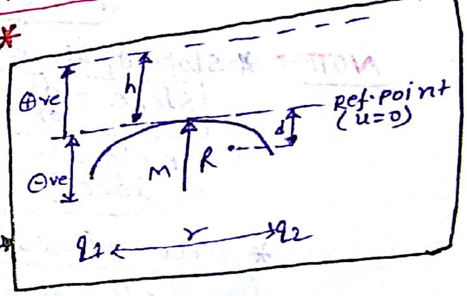


$$\boxed{U = \frac{1}{2}kx^2 = \frac{F^2}{2K} = -\frac{1}{2}Fx}$$

$$\boxed{U = \frac{1}{2}K\left(x_2^2 - x_1^2\right)}$$

|ii| → Gravitational potential Energy

$$\boxed{\begin{array}{ll} U_h = mgh & (h \ll R) \\ U_d = -mgd & (d \ll R) \end{array}}$$



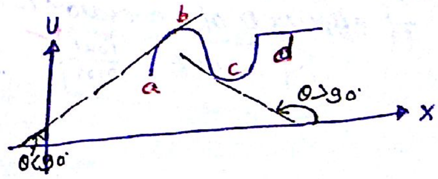
|iii| → Electrostatic P.E

$$U = \frac{Kq_1q_2}{r}$$

* Attractive force field $F = \ominus ve$, $U = \oplus ve$]
* Repulsive force field = $F = \oplus ve$, $U = \oplus ve$]

NOTE → P.E is Relative quantity & Its value depend on Reference point. (At Reference point P.E consider zero.)

Potential curve & Equilibrium condition



|a| → $a < x < b \Rightarrow$ slope $\Rightarrow \frac{+du}{dx} = \oplus ve \Rightarrow F = -\frac{du}{dx} = \ominus ve \Rightarrow$ Attractive force.

$\Rightarrow$ slope $\Rightarrow \frac{du}{dx} \Rightarrow \ominus ve \Rightarrow F = -\frac{du}{dx} \Rightarrow \oplus ve \Rightarrow$ Repulsive force.

|b| → $b < x < c \Rightarrow$ slope

|c| → At point b, c, d $\Rightarrow \frac{du}{dx} = 0 \Rightarrow F = 0 \Rightarrow$ Equilibrium cond^n.

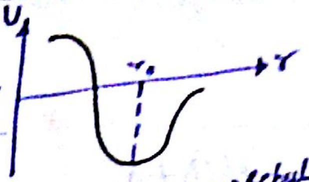
Equilibrium

Stable	Unstable	Neutral
* $F=0$	* $F=0$	* $F=0$
* $\frac{dU}{dr}=0$	* $\frac{dU}{dr}=0$	* $\frac{dU}{dr}=0$
* $\frac{d^2U}{dr^2}=\oplus ve$	* $\frac{d^2U}{dr^2}=\ominus ve$	* $\frac{d^2U}{dr^2}=0$
* U_{min}	* U_{max}	* $U=$ const.

(iv) → Intermolecular Potential Energy

$$\boxed{U=\frac{A}{r^{12}}-\frac{B}{r^6}}$$

$U=$ I.M pot. energy
$r=$ I.M. distance
$A, B=$ const.



* $r<r_0 \Rightarrow$ slope $\Rightarrow \frac{dU}{dr}=\ominus ve \Rightarrow F=-\frac{dU}{dr}=\oplus ve \rightarrow$ Repulsive force.
* $r>r_0 \Rightarrow$ slope $\Rightarrow -\frac{dU}{dr}=\oplus ve \Rightarrow F=-\frac{dU}{dr}=\ominus ve \Rightarrow$ Attractive force.
* $r=r_0 \Rightarrow$ slope $\Rightarrow \frac{dU}{dr}=0 \Rightarrow F=-\frac{dU}{dr}=0 \Rightarrow$ eqm point.

Work Energy theorm / Law of Mechanical Energy conservation

Net Workdone by any force is equal to change in K.E

$$[W_c+W_{N.C}+W_{ex}=\Delta K.E]$$

$$W_c=-\Delta U$$

$$[-\Delta U+W_{N.C}+W_{ex}=\Delta K.E+\Delta U] = (K.E_f-K.E_i)+(U_f-U_i)$$

* $W_{NC}=0 \Rightarrow W_{ex}=\Delta K.E+\Delta U$
* $W_{ex}=0 \Rightarrow W_{N.C}=\Delta K.E+\Delta U$
* $W_{N.C}=0 \Rightarrow \Delta U=0 \Rightarrow W_{ex}=\Delta K.E$
* $W_{NC}=0 \Rightarrow \Delta K.E=0 \Rightarrow W_{ex}=\Delta U.$
* $W_{NC}=0,\ W_{ex}=0 \Rightarrow \boxed{0=\Delta K.E+\Delta P.E}$

$$0=(K.E_f-K.E_i)+(U_f-U_i)$$

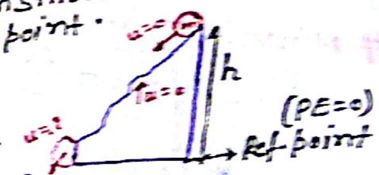
$$\boxed{K.E_i+U_i=K.E_f+U_f}$$

$$\boxed{K.E+U=\text{const.}} \Rightarrow \text{Law of mechanical Energy conservation.}$$

** Standard:

$U_f=0,\ a_R=$ const.
$t_1=t_2=t_3=----=$ same
$----9:7:5:3:1$

Particle is drop from height 'h' & it will move along the given track as shown. on smooth surface. velocity of particle At bottom point.



$W_{ex}+W_{N.C}=\Delta K.E+\Delta P.E$

$\boxed{K.E_i+U_i=K.E_f+U_f} \rightarrow$ M.E conservation.

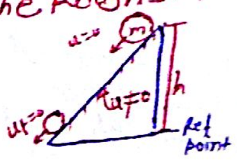
$0+mgh=\frac{1}{2}mV_f^2+0$

$$\boxed{V_f=\sqrt{2gh}}$$

Particle is drop height 'h' along the Rough surface of incline plane & Final velocity of particle at bottom point is zero. Than work done to got the mass bottom to top along the Rough surface.

* Top to bottom

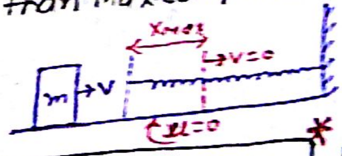
$$\boxed{W_{N.C}=-mgh}$$



* Bottom to top

$$\boxed{W_{ex}=2mgh}$$

A object of mass 'm' strike with velocity 'v' to the unstreachable string in a horizontal smooth surface. If force cofficient of spring is 'k' than max compression in spring.



$$\boxed{X_{max}=\left(\sqrt{\frac{m}{K}}\right)V}$$

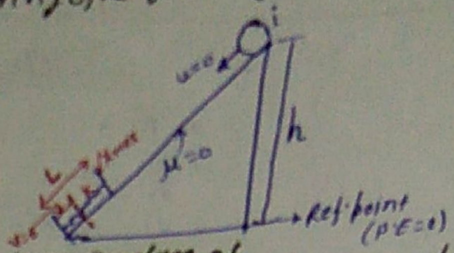
* If Friction coefficient of surface is 'μ'

$$\boxed{X_{max}=-\frac{\mu mg}{K}+\sqrt{\left(\frac{\mu mg}{K}\right)^2+\frac{M}{K}v^2}}$$

Particle of mass 'm' is drop from height 'h' along the surface of Incline plane. It strike at bottom point to the unstroachable spring of length 'L' ($L<<h$) then max compression. (force coefficient is 'k')

$W_{ex} = 0 = W_{N.C}$

$$\boxed{X_{max} = \sqrt{\frac{2mgh}{k}}}$$ *

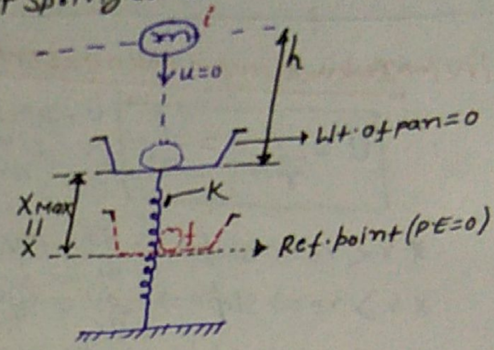


Object of mass 'm' is drop from height 'h' above from surface of Weightless pan. It strike at surface of pan & both are moves downward. If maximum compression in spring is 'X' than value of spring const

$W_{ex} = W_{N.C} = 0$

$K.E_i + U_i = K.E_f + U_f$

$$\boxed{K = \frac{2mgh(h+X)}{X^2}}$$ *

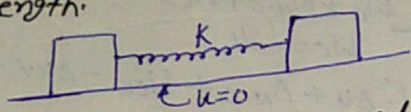


* Spring const is 'k' than max compression in length of spring.

$$\boxed{X = \frac{mg}{k} \pm \sqrt{\left(\frac{mg}{h}\right)^2 + \frac{2mgh}{k}}}$$ *

Two object of same mass is attach with spring & placed on smooth Horizontal surface. Initially spring present on natural length. If mass extend with external force. & Maximum extension in spring is 'X' than velocity of object when spring again Reach at its natural length.

$$\boxed{V = \sqrt{\frac{k}{2m}}\,X}$$ *



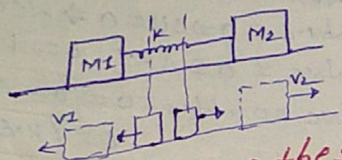
Two object of mass M_1 & M_2 attached with spring of const 'k'. It compress & Release than velocity of both mass when spring Reached at its natural length.

For M1

$$\boxed{V_1 = \sqrt{\frac{M_2}{M_1}\,\frac{kx^2}{M_1+M_2}}}$$

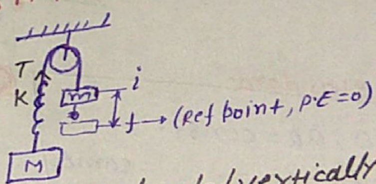
For M2

$$\boxed{V_2 = \sqrt{\frac{M_1}{M_2}\,\frac{kx^2}{M_1+M_2}}}$$



Min value of 'm' when it is Release. then contact of mass 'M' Release/Leave the surface.

$$\boxed{m = \frac{M}{2}}$$ *



* If whole length is string not spring. than $\boxed{m = M}$ *

Mass 'M' is attach with spring const 'k' in its natural length & It is handed vertically. when it is carfully Removed.

(i) → Extension in length of spring $\boxed{X_0 = \frac{Mg}{k}}$ *

(ii) → Maxm Extension in spring. $\boxed{X_{max} = \frac{2Mg}{k}}$ *

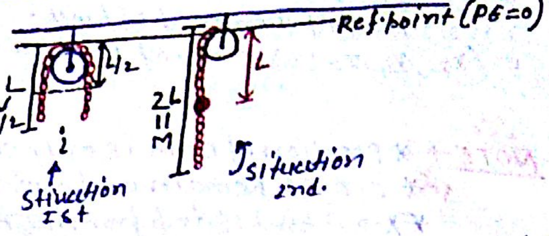
(iii) → If it will oscillate in vertical plane. Amplitude its oscillation.

$$\boxed{a = \frac{Mg}{k}}$$ *

A block of mass 'M' attach with spring const. 'k' in its natural length & placed on Rough horizontal surface. When mass 'M' move towards wall at distance 'a' & ~~length~~ Release than spring max. extended 'b' from ~~before~~ its natural length in opposite direction. than change in Amplitude in Half oscillation.

$$\boxed{a - b = \frac{2\mu Mg}{k}}$$

A chain of length '2L' mass 'M' is vertically hanged with of pin in equal vertical length. (From centre point). If one end is slightly disturb than chain move vertically downward than velocity of its lower end when second end reached at top point.



$W_{ext} + W_{N.C} = \Delta K.E + \Delta U$

$\boxed{KE_i + U_i = K.E_f + K.E_f}$

$0 + [(\frac{M}{2})(g)(-\frac{L}{2}) + (\frac{M}{2})(g)(-\frac{L}{2})]$

$= 2[(\frac{M}{2})g(\frac{L}{2})] = \frac{1}{2}mv^2 - MgL$

$= -\frac{MgL}{2} = \frac{1}{2}Mv^2 - MgL$

$= \frac{1}{2}Mv^2 = \frac{MgL}{2}$ *

$\boxed{V = \sqrt{Lg}}$

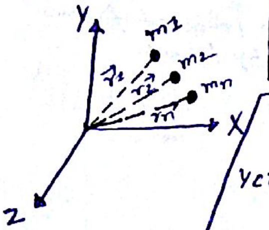
A chain of length 'L' mass 'M' present in a horizontal table. If $\frac{1}{n}$ part of its length is vertically handed from table find out work done to complete length on a horizontal table.

(i) → If surface is smoth → $\boxed{W_{ex} = \frac{MgL}{2n^2}}$ *

(ii) → Surface is Rough (μ) → $\boxed{W_{ex} = \frac{MgL}{2n^2}(1+\mu)}$ *

C.O.M

C.O.M of point mass system



$\boxed{\vec{r}_{COM} = \frac{m_1\vec{r_1} + m_2\vec{r_2} + m_n\vec{r_n}}{m_1 + m_2 + \cdots m_n}}$

$X_{cm} = \frac{m_1x_1 + m_2x_2 + \cdots m_nx_n}{m_1 + m_2 + \cdots m_n}$

$Y_{cm} = \frac{m_1y_1 + m_2y_2 + \cdots m_ny_n}{m_1 + m_2 + \cdots m_n}$

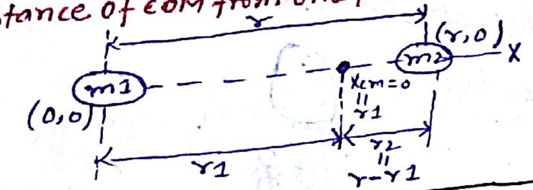
$Z_{cm} = \frac{m_1z_1 + m_2z_2 + \cdots m_nz_n}{m_1 + m_2 + \cdots m_n}$

*concept

$(X_1, Y_1, Z_1) \xrightarrow{d} (X_2, Y_2, Z_2)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

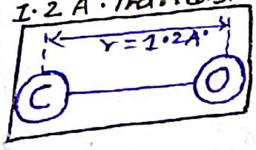
Distance of COM from one point mass in a two mass system.



$X_{CM} = \frac{m_1(0) + m_2(r)}{m_1 + m_2} = \frac{m_2 r}{m_1 + m_2} = r_1$

* Distance of C.O.M from mass $m_1 \Rightarrow r_1 = \frac{m_2}{m_1+m_2}$
* Distance of C.O.M from mass $m_2 \Rightarrow r_2 = r - r_1 = \frac{m_1 r}{m_1 + m_2}$

* Distance of C.O.M from C-atom. Intermolecular distance b/w C−O in a CO_2 is 1.2 Å. Than distance of com from C-atom.



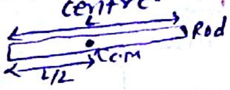
$M_w gm = N_A$
$1 = m_w\ amu$
$m_o = 16\ amu$
$m_c = 12\ amu$

$\boxed{r_c = \frac{m_o r}{m_c + m_o}}$ $= \frac{(16)(1.2)}{12+16} = 0.68$ Å

COM of continious mass distributed object.

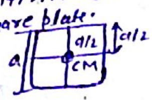
NOTE → If mass is uniformally distributed in a continious mass system then C.O.M is present its own centre.

(i) →

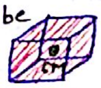


(ii) →

(iii) → square plate.



(iv) → cube



If mass is non-uniform

$\boxed{X_{CM} = \frac{\int (dm)x}{\int dm},\ Y_{CM} = \frac{\int (dm)y}{\int dm},\ Z_{CM} = \frac{\int (dm)z}{\int dm}}$

(a) → Linear object (1D) ⇒ λ ⇒ Linear mass density = $\frac{Mass}{length} = \frac{dm}{dl}$

(b) → 2-D object ⇒ σ ⇒ surface mass density = $\frac{Mass}{Area} = \frac{dm}{dA}$

(c) → 3-D object ⇒ ρ ⇒ volume mass density = $\frac{Mass}{vol} = \frac{dm}{dv}$

AMU
** concept

* If body not perform Rotational motion then force is applied its own C.O.M.

$(r_\perp = 0,\ \tau = 0)$

$(r_\perp \times F = 0)$

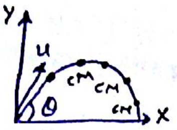
C.O.M of Remaning part

* m_1 = mass of complete object
* $(x_1, y_1, z_1) \Rightarrow$ C.O.M of complete object.
* m_2 = mass of Removed object.
* $(x_2, y_2, z_2) \Rightarrow$ C.O.M of Removed object.

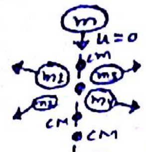
*
$$\boxed{\begin{aligned} X_{CM} &= \frac{m_1x_1 - m_2x_2}{m_1 - m_2} \\ Y_{CM} &= \frac{m_1y_1 - m_2y_2}{m_1 - m_2} \\ Z_{CM} &= \frac{m_1z_1 - m_2z_2}{m_1 - m_2} \end{aligned}}$$

***NOTE → * Position of COM is only change with external force.
Imp * C.O.M Remain unchange with Internal forces.
*Ex → A bomb is drop from height. If it is explot then direction of COM Remain unchange.

* → Y

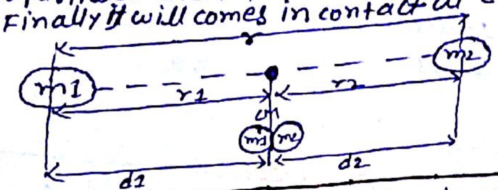


* →



मतलब आपस का मामला है उनलोग का मैंने कुछ भी नहीं किया !!

* Two particle move in mutual Internal force than position of COM Remain unchange & Finally it will comes in contact at C.O.M.



*
$$\boxed{\begin{aligned} &\text{* Distance covered by mass 'm1'} = d_1 = r_1 = \frac{m_2 r}{m_1+m_2} \\ &\text{* Distance covered by mass 'm2'} = d_2 = r_2 = \frac{m_1 r}{m_1+m_2} \end{aligned}}$$

A boat of Mass 'M' ⊕nt at Rest in a still water person of mass 'm' moves in boat from one end to another end. Length of Its path 'L'. than distance covered by boat If person Reached one end to another end.

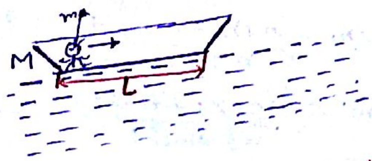
$r_{cm} = 0 \Rightarrow \dfrac{m_1\vec{r_1} + m_2\vec{r_2}}{m_1+m_2}$

$\boxed{m_2\vec{r_2} = -m_1\vec{r_1}}$

* $(m_p + m_b)\vec{r_B} = -m_p\vec{r_P}$

$\vec{r_B} = -\left(\dfrac{m_p}{m_p+m_b}\right)\vec{r_P}$

*
$$\boxed{\vec{r_B} = -\left(\frac{m}{m+M}\right)(L)}$$



Person of mass 'm' moves in upward direction than disp. of ballon in downward direction.

$m_1r_1 = -m_2r_2$

$m_p\vec{r_p} = -(m_p + m_b)\vec{r_b}$

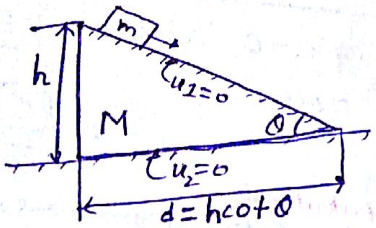
*
$$\boxed{\vec{r_b} = -\left(\frac{m}{m+M}\right)(L)}$$

* $\ominus \Rightarrow$ Indicated downward coming.

Horizontal dist. covered by mass 'M' when 'm' Reached from top to bottom along the smoth Incline plane.

$\boxed{m_1\vec{r_1} = -m_2\vec{r_2}}$

*
$$\boxed{\vec{r_2} = -\frac{m(h\cot\theta)}{m+M}}$$



Velocity & Acceleration of COM

$$\boxed{\begin{aligned} \vec{V}_{C.M} &= \frac{M_1\vec{V_1} + M_2\vec{V_2} - - - - - - - M_n\vec{V_N}}{M_1 + M_2 - - - - - - M_N} \\ \vec{a}_{C.M} &= \frac{M_1\vec{a_1} + M_2\vec{a_2} - - - - M_N\vec{a_n}}{M_1 + M_2 + - - - - - - M_N} \end{aligned}}$$

* Standard:

If M1 = M2

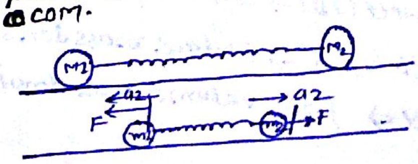
$\boxed{\vec{a}_{CM} = 0}$

Two Mass M1 & M2 connected with spring & it is placed on smooth horizontal surface. It is compresed & Release than accleration of COM.

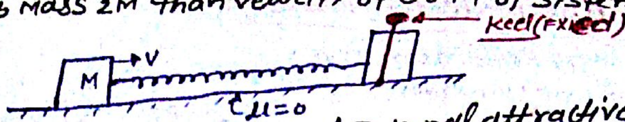
$\vec{a}_{CM} = \dfrac{m_1a_1 + m_2(-a_2)}{m_1+m_2}$

$F = \text{same} = m_1a_1 = m_2a_2$

$\boxed{\vec{a}_{C.M} = 0}$ *

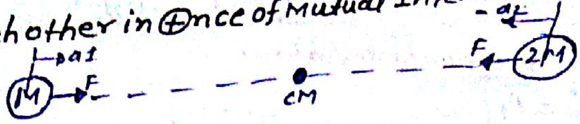


If mass 'M' moves with speed 'V' towards mass 2M than velocity of COM of system at that instant.



$$V_{CM} = \frac{m(V) + 2M(0)}{M+2M} = \frac{V}{3}$$

Two mass m & 2M moves toward each other in presence of mutual internal attractive force. than Accln of COM.



F = same $= M(a_1) = 2M(-a_2)$

$$\vec{a}_{CM} = \frac{M(a_1) + 2M(-a)}{M+2M} = 0$$

$\boxed{V_{C.M} = 0}$

Linear Momentum conservation

If Ext. Force act on system is zero. than linear momentum of system is conserve.

* It can't change with internal forces.

$\boxed{F_{en} = \frac{dP}{dt} = 0}$ $\boxed{dP = 0}$ $\boxed{\vec{P} = const.}$

$\boxed{\vec{P_i} = \vec{P_f}}$

Ex → * When bullet is fired from Gun is follow linear momentum conservation. Gun Require in the opposite direction of bullet.

* When α-particle is emmited from nucleus External force on system is zero thats why linear momentum conserve.

Nucleus Z^{X^A} emit α-particle with speed 'V' than –

(i) → Require velocity of Remaning nucleus (Daughter nucleus)

(ii) → If Release energy is 'Q' than K.E of α-particle & Daugher nucleus.

$$Z^{X^A} \longrightarrow {}_2He^4 \;(\alpha\text{-particle}) + Z\text{-}2^{Y^{A-4}} + Q$$

He-nucleus (Daughter Nucleus)

***** (i) → $Z\text{-}2^{Y^{A-4}} \equiv Z\text{-}2^{X^A} \longrightarrow {}_2He^4 \rightarrow V_\alpha = V$

$\boxed{Z^{X^A} \rightarrow \text{mass of 1 nucleus} = A \text{ amu or Amp}}$

$F_{ext} = 0 \quad \vec{P_i} = \vec{P_f}$

$0 = \vec{P_\alpha} + \vec{P_\beta}$

$\vec{P_D} = \vec{P_\alpha}$

$\vec{P_D} = -\vec{P_\alpha}$

$\vec{P_D}\vec{V_D} = -M_\alpha \vec{V_\alpha}$

$\vec{V_D} = -\left(\frac{M_\alpha}{M_D}\right)\vec{V_\alpha}$

$\boxed{M_\alpha = 4\,a.m.u \quad M_D = (A-4)\,a.m.u}$

*** $\boxed{V_D = -\left(\frac{4}{A-4}\right)\vec{V_\alpha}}$

**** (ii) → When α-particle is emitted from nucleus Release energy distributed b/w α-particle & Daughter nucleus in form of K.E.

$\boxed{Q = \left(\frac{m_D}{m_\alpha}\right)K.E_D + K.E_D}$

$\boxed{M_\alpha = 4\,amu \quad M_D = (A-4)\,a.m.u \quad M_D + M_\alpha = 'A'\,a.m.u}$

**** $K.E_D = \left(\frac{m_\alpha}{m_D + m_\alpha}\right)Q$

$K.E_\alpha = \left(\frac{m_D}{m_D + m_\alpha}\right)Q$

AIIMS ***

$$\boxed{K.E_D = \left(\frac{4}{A}\right)Q \lll Q \qquad K.E_\alpha = \left(\frac{A-4}{A}\right)Q \approx Q}$$

COLLISION

- Direction
  - Head on callison
  - oblique callision
- Nature
  - Elast callision → Momentum & Energy both conserve.
  - Inelastic callison. → * Momentum is conserve. * energy not conserve.

Newton's Law callision

Ratio of Relative velocity after callision & before callison Remain same & equal to minus of Restitution coefficient.

* $V_2 - V_1$ → Relative velocity After callision.
* $U_2 - U_1$ → Relative velocity before collision.

$$\boxed{\frac{V_2 - V_1}{U_2 - U_1} = \text{same} = e}$$

* e = Restitution coefficent
  - Perfectelly Elastic callison ⇒ e = 1 (max)
  - Perfectelly Inelastic callison ⇒ e = 0
  - Ronge ⇒ $\boxed{0 < e < 1}$

NOTE → Newton law of callison is applicable along the line of Impact.

[A] → Head on perfectelly Elastic callison

* Energy Remain conserve.
* Linear Momentum conserve.
* $\boxed{e = 1}$ *

$\boxed{\Delta p = \text{same} = p = \text{same}}$

$\Delta p = Ft$, $\Delta p = F \cdot t$

|i| → Linear Momentum conservation.

$$\boxed{\vec{P_i} = \vec{P_f}}$$

$$m_1u_1 = m_2u_2 = m_1v_1 + m_2v_2 \quad \text{—(I)}$$

|ii| → $e = 1 \Rightarrow -1 = \frac{v_2 - v_1}{u_2 - u_1} \Rightarrow v_2 - v_1 = -(u_2 - u_1)$ —(II)

*

$$v_1 = \frac{2m_2u_2}{m_1 + m_2} + \frac{(m_1 - m_2)u_1}{m_1 + m_2}$$

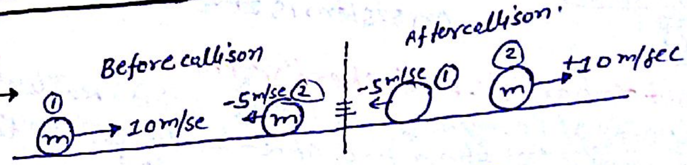
$$v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{(m_2 - m_1)u_2}{m_1 + m_2}$$

** Standard condition

BCECE 2015 → |a| → $m_1 = m_2$

$\boxed{v_1 = u_2}$
$\boxed{v_2 = u_1}$

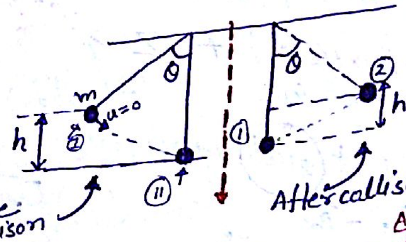
Ex →



|b| → $m_1 = m_2,\ u_2 = 0$

$\boxed{v_1 = u_2 = 0}$
$\boxed{v_2 = u_1}$

Ex →



** IIT |c| → $m_1 >>> m_2$

$\frac{m_2}{m_1} <<< 1$ ← neglet

* $\boxed{v_1 = 0 + \left(\frac{1 - 0}{1 + 0}\right)u_1}$

* $\boxed{v_2 = 2u_1 - u_2}$

$m_1 <<< m_2$

$\frac{m_1}{m_2} <<< 1$ ← neglect.

* $\boxed{v_1 = 2u_2 - u_1}$

* $\boxed{v_2 = +u_2}$

AIIMS
$m_1 <<< m_2 \Rightarrow u_2 = 0$

$\boxed{v_1 = -u_1}$
$\boxed{v_2 = u_2 = 0}$

Ex → * gass molecules & container Wall.

Transfered Energy in a perfectelly Elastic callison. Transfered from Ist particle to IInd particle.

|i| → K·E Transfered from Ist particle to IInd particle.

$$\boxed{\frac{\Delta K \cdot E_f}{K \cdot E_1} = 1 - \left(\frac{v_1}{u_1}\right)^2}$$

|ii| → Remaning fractional change in K·E of Ist particle

$$* \ \boxed{\left|\frac{\Delta K \cdot E_{remain}}{K \cdot E_1}\right| = \left(\frac{v_1}{u_1}\right)^2 = 1 - \left|\frac{\Delta K \cdot E_1}{K \cdot E_1}\right|}$$

$u_2 = 0$ (Perfectelly Elastic Head on callision) *

$$\boxed{\frac{v_1}{u_1} = \frac{m_1 - m_2}{m_1 + m_2}}$$

|i| → Transfered Energy from Ist particle to 2nd particle.

$$\frac{\Delta K \cdot E_1}{K \cdot E_1} = \frac{4m_1m_2}{(m_1 + m_2)^2} = \frac{4m_1m_2}{(m_1 - m_2)^2 + 4m_1m_2}$$

$$\Downarrow$$

$$\frac{4\left(\frac{m_2}{m_1}\right)}{1 + \left(\frac{m_2}{m_1}\right)^2}$$

AIIMS **

|a| → Transfer Energy is max When $m_1 = m_2 \Rightarrow \frac{\Delta K \cdot E}{K \cdot E_2} = 1\ (100\%)$

|b| → $\frac{m_2}{m_1} \uparrow$ Ratio $\Longrightarrow$ Fractional transfered Energy $\downarrow$

AIIMS

A→ # Why Heavy Water is best Moderator !

Reason → When comparable mass body collide elastically Maximum energy transfer takes place. & Mass of neutron is in order of Mass of Deutron that's why Max energy transfer take place.

That's why H_2O not best Moderator.

Neutron $_0n^1$ (m_1)	Moderator (m_2)	$\frac{\Delta K.E_1}{K.E_1}$	Remaning energy
1 a.m.u	$_1H^1$ = 1 a.m.u	$\boxed{100\%}$	0
1 a.m.u	$_1H^2$ = 2 a.m.u	$\frac{4(1)(2)}{(1+2)^2} = \boxed{\frac{8}{9}}$ (AIIMS)	$\frac{1}{9}$
1 a.m.u	$_6C^{12}$ = 12 a.m.u	$\frac{4(1)(12)}{(1+12)^2} = \frac{98}{169}$	
1 a.m.u	$_8O^{16}$ = 16 amu	$\frac{4(1)(16)}{(1+16)^2}$	
	Ba > 100 amu		

Remaning Fractional energy. ($u_2=0$)

$$\boxed{\left|\frac{\Delta K_{Remain}}{K.E_1}\right| = \left(\frac{v_1}{u_1}\right)^2 = \left(\frac{m_1-m_2}{m_1+m_2}\right)^2}$$

$u_2=0$ (Perfectelly Elastic callision)

* $\frac{\Delta K.E_{Transfered}}{K.E} = \frac{4m_1m_2}{(m_1+m_2)^2}$

* $\frac{\Delta K.E_{Remaning}}{K.E} = \left(\frac{m_1-m_2}{m_1+m_2}\right)^2$

[B] → Inelastic Headon callision

* Energy Loss
* Linear Momentum Remain conserve
* $0<e<1$

Linear Momentum conservation.

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ —(I)

$-e = \frac{v_2-v_1}{u_2-u_1} \Rightarrow v_2 - v_1 = -e(u_2-u_1)$ —(II)

$$\vec{v_1} = \frac{(e+1)m_2\vec{u_2}}{m_1+m_2} + \left(\frac{m_1-em_2}{m_1+m_2}\right)\vec{u_1}$$

$$\vec{v_2} = \frac{(e+1)m_1\vec{u_1}}{m_1+m_2} + \left(\frac{m_2-em_1}{m_1+m_2}\right)\vec{u_2}$$

Loss in K.E

overall loss

$\Delta K.E_{Loss} = \frac{1}{2}\left(\frac{m_1+m_2}{m_1+m_2}\right)(\vec{u_1}-u_2)(1-e^2)$

|a| → Same direction.

$\Delta K.E_{Loss} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1-u_2)^2(1-e^2)$

|b| → Diff or opposite direction

$\Delta K.E_{Loss} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1+u_2)^2(1-e^2)$

|C| → Perfectelly Inelastic Headon callision

* Energy Loss (Max)
* $e = 0 = -1\left(\frac{v_2-v_1}{u_2-u_1}\right) \Rightarrow v_2 = v_1$
* Linear Momentum conserved.
* Velocity of both particle after callision become same.

$m_1+m_2 = m_1v_f + m_2v_f$

$$\boxed{v_f = \frac{m_1\vec{u_1}+m_2\vec{u_2}}{m_1+m_2}}$$

$\boxed{\Delta K.E_{loss} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(\vec{u_1}-\vec{u_2})(1-0^2)}$

|a| → same direction

$v_f = \frac{m_1u_1+m_2u_2}{m_1+m_2}$

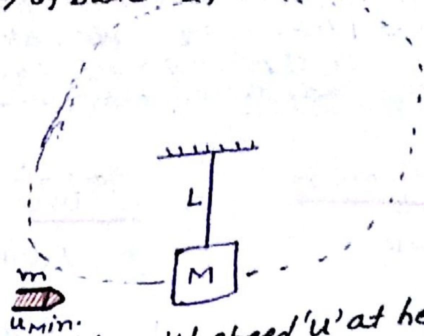
$\Delta K.E_{loss} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1-u_2)^2$

|b| → opposite direction

$v_f = \frac{m_1u_1-m_2u_2}{m_1+m_2}$

$\Delta K.E_{loss} = \frac{1}{2}\left(\frac{m_1m_2}{m_1+m_2}\right)(u_1+u_2)^2$

Bullet of mass 'm' perform Inelastic callision of bag of mass 'M' which is suspeded with string of length 'L' than minimum velocity of bullet : If after callison combination perform circular motion in vertical plane.

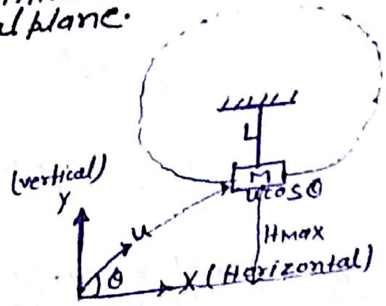


$$\boxed{V_c = \frac{mu}{M+m}}$$

* condition to complete vertical circular motion.

$$\boxed{V_c \geq \sqrt{5Lg}} \qquad \boxed{U_{min} = \frac{M+m}{m}\sqrt{5Lg}} \; *$$

Particle of mass 'm' projected at angle 'θ' from horizontaly with speed 'u' at heighest point of its path. It perform perfectelly inelastic callision with object mass 'M' wich is suspended with string of length 'L' than minimum value of 'u' If combination perform circular motion in a vertical plane.



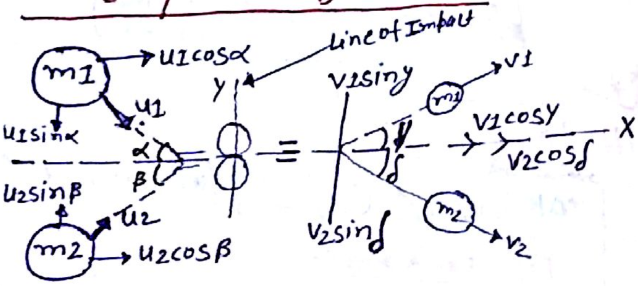
$$\boxed{V_c = \frac{m(u\cos\theta)}{M+m}}$$

* $V_c \geq \sqrt{5Lg}$

$$\boxed{u \geq \frac{M+m}{m\cos\theta}\sqrt{5Lg}}$$

$$\boxed{U_{min} = \frac{1}{\cos\theta}\left(\frac{M+m}{m}\right)\sqrt{5Lg}} \; **$$

|D| → oblique / Glancing collision →



** Standard.

Elastic oblique callision

$M_1 = M_2$
$e = 1$
$u_2 = 0$
$\Big\}$ $\alpha + \beta = 90°$

Ist condn

Linear Momentum conservation

$$\boxed{\vec{P}_{1xi} + \vec{P}_{2xi} = \vec{P}_{1xf} + \vec{P}_{2xf}} \text{ or } \boxed{\vec{P}_{1i} + \vec{P}_{2i} = \vec{P}_{1i} = \vec{P}_{1f}}$$

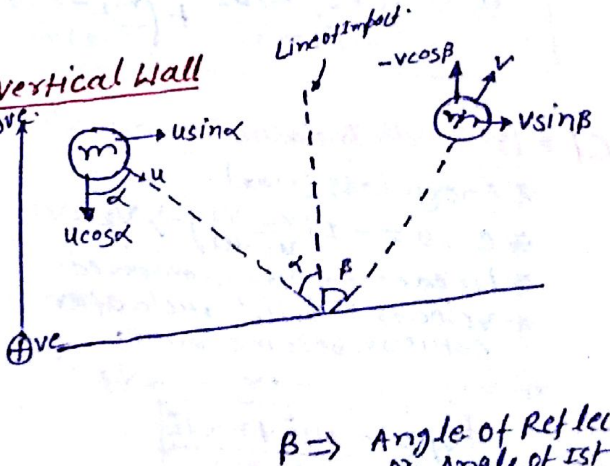
$$\boxed{\vec{P}_{1yi} + \vec{P}_{2yi} = P_{1yf} + P_{2yf}}$$

2nd condn

$$\boxed{-e = \frac{(v_2)_y - (v_1)_y}{(u_2)_y - (u_1)_y}}$$

|E| → callision of particle with Horizontal or, Vertical wall

u = velocity of Ist particle Before callision.
v = velocity of Ist particle After callision
α = Angle of Incidente
or, Angle of Ist particle from line of Impact.
or, Angle of Ist particle from Normal of surface.



$F_{||} = 0 \Rightarrow v = \text{const}$ *

$$\boxed{\vec{V_i} = \vec{V_f}} \; * \quad \boxed{u\sin\alpha = v\sin\beta} \text{ — (I)}$$

β ⇒ Angle of Reflection or, Angle of Ist particle after callision from Normal or, Line of Impact.

$-e = \left(\frac{v_2 - v_1}{u_2 - u_1}\right)_{\perp}$ direction (Normal to the surface)

*
$$\boxed{v\cos\beta = +eu\cos\alpha} \text{ — (II)}$$

|I| → $\frac{eq(I)}{eq(II)} \Rightarrow \frac{v\sin\beta}{v\cos\beta} = \frac{u\sin\alpha}{eu\cos\alpha}$.

$$\boxed{\tan\beta = \frac{1}{e}\tan\alpha} \; **$$

III → $(v\sin\beta)^2 + (v\cos\beta)^2 = (u\sin\alpha)^2 + (eu\cos\alpha)^2$

$v^2(\sin^2\beta + \cos^2\beta) = u^2(\sin\alpha + e^2\cos^2\alpha)$

$$\boxed{v = u\sqrt{\sin^2\alpha + e^2\cos^2\alpha}}\ *$$

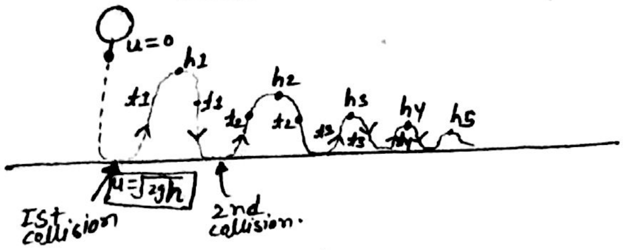
**

Callision perfectelly Elastic (e = 1)

* $\tan\theta = \frac{\tan\alpha}{1} \Rightarrow \boxed{\beta = \alpha}$ *
* $v = u\sqrt{\sin^2\alpha + (1)\cos^2\alpha} = \boxed{u}$ *

$\alpha = 0 \Rightarrow \tan\beta = \frac{\tan 0^\circ}{e} = \boxed{0}$ *

$\boxed{\beta = 0}$ *



Velocity After Ist callision

$\alpha = 0 \Rightarrow v = u\sqrt{\underbrace{\sin^2 0}_{0} + e^2\underbrace{\cos^2 0}_{1}}$

$\boxed{v = eu}$ *

Velocity of particle Ist callision → $\boxed{v_1 = eu}$ **

* velo After 2nd callision → $\boxed{v_2 = ev_1 = e^2u}$ *
* velo. After 3rd callision → $\boxed{v_3 = ev_2 = e^3u}$

⋮

velo. after nth callision → $\boxed{v_n = ev_{(n+1)} = e^n u}$

Max Height attain by particle.

$\boxed{h_{max} = \frac{u^2}{2g}}$

* After Ist callison $h_1 = \frac{v_1^2}{2g} = \frac{e^2u^2}{2g} = \frac{e^2(\sqrt{2gh})^2}{2g} = \boxed{e^2h}$ *
* After 2nd callision $h_2 = e^2h_1 = \boxed{e^4h}$
* " 3rd " $h_3 = e^2h_2 = \boxed{e^5h}$
* " nth " $h_n = e^2h_{n-1} = \boxed{e^{2n}h}$ *

Time to attain Max. height.

$T = t_0 = \sqrt{\frac{2H}{g}} = t_1 = \sqrt{\frac{2H_1}{g}} = \sqrt{\frac{2(e^2H)}{g}} = \boxed{e\sqrt{\frac{2H}{g}}}$ *

$\boxed{t_1 = et_0}$

Time to attain b/w bottom to top, top to bottom

Ist callision → $T_1 = \boxed{et_0}$ *

2nd callision → $T_2 = eT_1 = \boxed{e^2t_0}$

nth callision → $T_n = eT_{n-1} = \boxed{e^n t_0}$

Distance covered by particle before comes to Rest.

$$\boxed{Dist(D) = h\left(\frac{1+e^2}{1-e^2}\right)}$$

Time taken by particle comes to Rest.

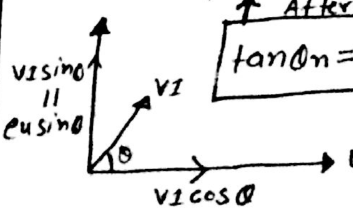
$$\boxed{t = t_0\left(\frac{1+e}{1-e}\right) = \left(\frac{1+e}{1-e}\right)\sqrt{\frac{2H}{g}}}$$

*! only for Adult

***** !!

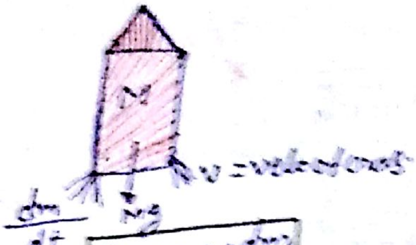
To Find angle change After callision. *

$$\boxed{\tan\theta_n = e^n\tan\theta}$$



Motion of Rocket

$$\boxed{-v\frac{dm}{dt} = M(g+a)}$$



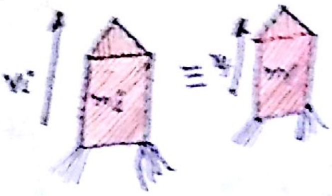
$$\boxed{F_{gas} = v\frac{dm}{dt}}$$

velocity of Rocket const

$$\boxed{a=0}$$

$$\boxed{-v\frac{dm}{dt} = Mg}$$

Rocket Move with const. Accln



$$-v\frac{dm}{dt} = Mg + Ma$$

$$-d\frac{vdm}{dt} = Mg + M\frac{dv_r}{dt}$$

$$-\frac{v\,dm}{M} = g\,dt + dv_r$$

$$\boxed{\int_{v_i}^{v_f} dv_r = -g\int_0^t dt = v\int_{M_i}^{M_f}\frac{dm}{M}}$$

$$\boxed{v_f = v_i - gt_f - v\log\left(\frac{M_f}{M_i}\right)}$$

If start from Rest ($v_i = 0$), Effect of Gravity is negligible ($gt_f = 0$)

$$\boxed{v_f = -v\log_e\left(\frac{m_f}{m_i}\right) = -2.303\,v\log_{10}\left(\frac{m_f}{m_i}\right)}$$

velocity of Rocket